大物

向量处理

解析：

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}.$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

矢量不能直接积分，
可以先投影到x,y,z轴，
分别进行积分，再矢量合成

$$\frac{d\vec{A}}{dt} = \frac{d}{dt}(A_x\vec{i} + A_y\vec{j} + A_z\vec{k})$$
$$= \frac{dA_x}{dt}\vec{i} + \frac{dA_y}{dt}\vec{j} + \frac{dA_z}{dt}\vec{k}$$

eg. $\vec{F}\cdot d\vec{r} = -G\frac{Mm}{r^2}\cdot\frac{\vec{r}}{r}d\vec{r}$

圆周运动

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{\tau}) = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt}$$

沿切向($\hat{\tau}$)
即切向加速度

$$\vec{a_\tau} = \frac{dv}{dt}\hat{\tau} = \frac{d^2s}{dt^2}\hat{\tau}$$

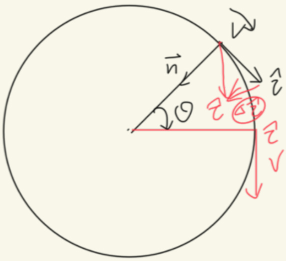
沿法向($\vec{n}$)
即法向加速度

$$\vec{a_n} = v\frac{d\hat{\tau}}{dt} = \frac{v^2}{\rho}\vec{n}.$$

横向对比

匀直	圆周
$x = x_0 + vt$	$\theta = \theta_0 + \omega t$
$v = v_0 + at$	$\omega = \omega_0 + \beta t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega t + \frac{1}{2}\beta t^2$
$v^2 - v_0^2 = 2ax$	$\omega^2 - \omega_0^2 = 2\beta\theta$

推导过程.



$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d(V\hat{\imath})}{dt} = \frac{dV}{dt}\hat{\imath} + \frac{d\hat{\imath}}{dt}V$$

对于 $\vec{V} = V\cdot\hat{\imath} = \frac{ds}{dt}\hat{\imath}$

$$\therefore \frac{dV}{dt}\hat{\imath} = \frac{d\frac{ds}{dt}}{dt}\hat{\imath} = \frac{d^2s}{dt^2}\hat{\imath}$$

$$\frac{d\hat{\imath}}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\hat{\imath}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\theta\cdot\hat{n}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\rho\Delta\theta\hat{n}}{\rho\cdot\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta s\,\hat{n}}{\Delta t\cdot\rho} = \frac{1}{\rho}V\cdot\hat{n}$$

$$\therefore \frac{d\hat{\imath}}{dt}V = \frac{V^2}{\rho}\hat{n}.$$

$$\vec{r} = (2t)\vec{i} + (3t^2+4)\vec{j}$$

$$a_\tau = \frac{dv}{dt} =$$

运动方程:

$$\begin{matrix} \vec{r}(t) \\ s(t) \\ \theta(t) \end{matrix} \Rightarrow \vec{v}(t) \Rightarrow \vec{a}(t) = \begin{cases} a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \\ a_\tau\vec{\tau} + a_n\vec{n} \end{cases}$$

$$\vec{v}(t) = \frac{ds}{dt}\vec{\tau}$$

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

$$\bar{\vec{v}} = \frac{\Delta r}{\Delta t} = \frac{\vec{r} - \vec{r}_0}{\Delta t}$$

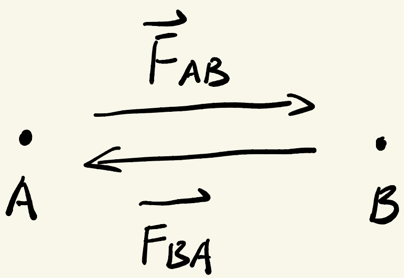
解题流程

1、列出运动方程 $\begin{cases} \vec{r}(t) \\ s(t) \\ \theta(t) \end{cases}$

2、求导，得出 $\vec{v}(t) / \omega(t)$

3、寻找关系 $\begin{cases} \text{初始量} \\ \text{牵连速度} \end{cases}$

4、求解



$$\bar{F}(t) = m\frac{dv}{dt}$$

$$\bar{F}(v) = m\frac{dv}{dt}$$

$$\bar{F}(x) = m\cdot\frac{dv}{dx}\cdot\frac{dx}{dt}$$

质点的动量定理

力的时间积累效应 ⟺ 物体动量变化

$$\int_{t_0}^{t}\vec{F}\,dt = \vec{p} - \vec{p}_0 = m\vec{v} - m\vec{v}_0$$

平均冲力：$$\bar{\vec{F}} = \frac{\int_{t_0}^{t}\vec{F}\,dt}{t-t_0} = \frac{m\vec{v} - m\vec{v}_0}{t-t_0}$$

质点系：$$\int_{t_0}^{t}\left(\sum \vec{F}_i\right)dt = \overbrace{\left(\sum m_i\vec{v}_i\right)}^{\vec{p}_i} - \overbrace{\left(\sum m_i\vec{v}_{i0}\right)}^{\vec{p}_{i0}}$$

力的瞬时作用规律：$$\vec{F} = \frac{d\vec{p}}{dt}$$

火箭飞行:

(变质量问题)

$$mv = (m+\underbrace{dm}_{})(v+dv) - dm(v+dv-u)$$

dm 小于0, 有符号

$$mdv + udm = 0$$

即 $$dv = -u\frac{dm}{m}$$

$$\int_{v_1}^{v_2} dv = -\int_{m_1}^{m_2} u\frac{1}{m}dm$$

$$v_2 - v_1 = u\ln\frac{m_1}{m_2}$$

$$v = \boxed{u}\cdot\ln\boxed{\frac{m_0}{m}}$$

喷气速度　　质量比

$$\boxed{m\frac{d\vec{v}}{dt} = \vec{F} + \vec{v}_r\frac{dm}{dt}}$$

密歇尔方程.

动能（状态函数）

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

适用于惯性系

质点的动能定理.

$$A = \int_a^b \vec{F}\cdot d\vec{r} = \int_{v_a}^{v_b} d\left(\frac{1}{2}mv^2\right) = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2 = E_{kb} - E_{ka}$$

质点系 $\Rightarrow A_{外} + A_{内} = E_k - E_{k0} = \Delta E_k$

保守力：

特征：$A = \oint_L \vec{F}\cdot d\vec{r} = 0$

机械能守恒：只有保守内力作功情况下

角动量

右手螺旋

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r}\,\vec{v}$$

$\vec{r} \to \vec{v}$

$$L = m \cdot v \cdot r\sin\varphi$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt}$$

$$= \vec{v}\times \overset{=0}{m\vec{v}} + \vec{r}\times\frac{d\vec{p}}{dt}$$

即 $\dfrac{d\vec{L}}{dt} = \vec{r}\times\vec{F} = \vec{M}$ [力矩]

$$\vec{M} = \vec{r}\times\vec{F} = \frac{d\vec{L}}{dt}$$ 质点的角动量定理.

内力的力矩矢量和为零. $\sum \vec{M}_{内} = 0$

$$\int_0^t \vec{M}\,dt = \vec{L} - \vec{L}_0$$

角量与线量的关系:

$$\boxed{\vec{a} = r\beta\hat{\tau} + r\omega^2\hat{n}}$$

角速度矢量.

$$\vec{v} = \vec{\omega} \times \vec{r}.$$

$$M_z = J\alpha = J\frac{d\omega}{dt}$$

由此引出转动惯量. $J = \sum r_i^2 \Delta m_i$

$$\boxed{J = \int r^2 dm}$$

平行轴定理:

$$\boxed{J = J_c + mh^2}$$

垂直轴定理:

$$J_z = J_x + J_y$$

体系对比

$\omega = \frac{d\theta}{dt}$	$v = \frac{ds}{dt}$
$\alpha = \frac{d\omega}{dt}$	$a = \frac{dv}{dt}$
$\theta = \omega t$	$s = vt$
$\omega = \omega_0 + \beta t$	$v = v_0 + at$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\beta t^2$	$x = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega^2 - \omega_0^2 = 2\beta(\theta - \theta_0)$	$v^2 - v_0^2 = 2a(x - x_0)$
$M = J\alpha$	$F = ma$
$Mt = J\omega - J_0\omega_0$	$Ft = mv - mv_0$
$\sum J\omega = C$	$\sum mv = C$
$E_K = \frac{1}{2}J\omega^2$	$E_K = \frac{1}{2}mv^2$
$A = M\theta$	$A = Fs$
$M\theta = \frac{1}{2}J\omega^2 - \frac{1}{2}J\omega_0^2$	$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

电场与电荷

引入新常量 ε_0

$$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$$

$$\vec{F}_{库} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \cdot \vec{e}_r$$

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \frac{1}{4\pi\varepsilon_0} \sum \frac{q q_i}{r_i^2} \vec{e}_{ri}$$

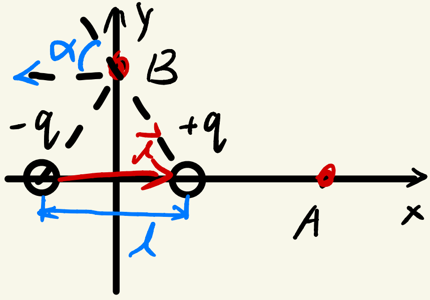
电场强度

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

点电荷：

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{e}_r}$$

$\boxed{\text{电偶极子}}$ $\vec{p} = q\vec{l}$



$$E_A = E_+ + E_- = \frac{2q x l}{4\pi\varepsilon_0[x^2-(\frac{l}{2})^2]^2}\vec{i}$$

由于 $x>>l$ 即 $\frac{l^2}{4x^2}<<1$

$$\therefore E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2ql}{x^3}\vec{i} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{p}}{x^3}.$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{p}}{x^3}$$

$$E_B = E_+ + E_- = -2\frac{q}{4\pi\varepsilon_0(y^2+\frac{l^2}{4})}\cos\alpha\,\vec{i} = -\frac{ql}{4\pi\varepsilon_0(y^2+\frac{l^2}{4})^{\frac{3}{2}}}\vec{i}$$

由 $y>>l$，即 $(y^2+\frac{l^2}{4})^{\frac{3}{2}}\approx y^3$.

$$\therefore E_B = -\frac{1}{4\pi\varepsilon_0}\frac{ql}{y^3}\vec{i} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{\vec{p}}{y^3}$$

$$E_y = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$$

受力情况：

$F = qE$ $\quad M = Fl\sin\theta = qEl\sin\theta = pE\sin\theta$

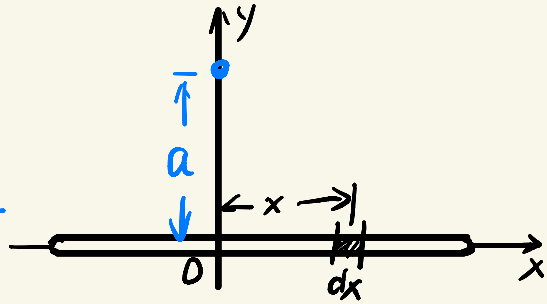
$$\therefore \vec{M} = \vec{p}\times\vec{E}$$

均匀带电直棒电场

电荷元 ⟶ 微元法　　$d\vec{E} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{dq}{r^2}\vec{e}_r$

$$\vec{E} = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\vec{e}_r$$

$\lambda = \dfrac{q}{L}$



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{r^2}\vec{e}_r$$

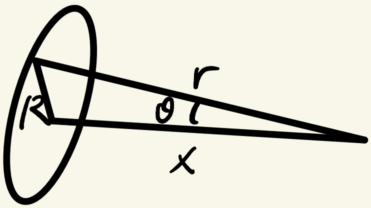
$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$

无限长，即 $L \to \infty$，

$$E = \frac{\lambda}{2\pi\varepsilon_0 a}$$

带电圆环



$$\lambda = \frac{q}{2\pi R}$$

$$E = E_x = \sum E\cos\theta$$

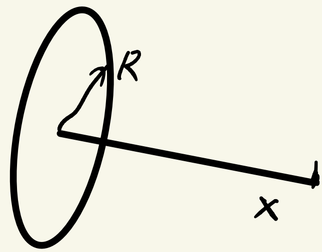
$$\therefore E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \frac{x}{r} = \frac{qx}{4\pi\varepsilon_0 (x^2+R^2)^{\frac{3}{2}}}$$

若 $x \gg R$

则

$$E = \frac{q}{4\pi\varepsilon_0 x^2}$$

带电圆盘



电荷面密度 σ

$$dq = \sigma 2\pi r dr$$

$$dE = \frac{x\,dq}{4\pi\varepsilon_0 (x^2+r^2)^{\frac{3}{2}}} = \frac{\sigma x}{2\varepsilon_0}\frac{r\,dr}{(x^2+r^2)^{\frac{3}{2}}}$$

带电圆环

$$E = \int dE = \frac{\sigma x}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{\frac{3}{2}}} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

(1) 若 $R \gg x$，则

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

无限大圆盘

(2) 若 $x \gg R$，则 $\left(1+\frac{R^2}{x^2}\right)^{-\frac{1}{2}} = 1-\frac{1}{2}\frac{R^2}{x^2}+\frac{3}{8}\left(\frac{R^2}{x^2}\right)^2-\cdots \approx 1-\frac{1}{2}\frac{R^2}{x^2}$

$$\vec{E} = \frac{\sigma R^2}{4\varepsilon_0 x^2}\vec{i} = \frac{q}{4\pi\varepsilon_0 x^2}\vec{i}$$

点电荷激发

电场强度通量

$$\Psi_E = E\cos\theta \cdot S = \vec{E} \cdot \vec{S}$$ 面积矢量

$$\Psi_E = \iint_S \vec{E}\, d\vec{S} \ / \ \oiint_S \vec{E} \cdot d\vec{S}$$

从曲面内向外穿出为正

从外向内穿入为负

Summary

$k = \frac{1}{4\pi\varepsilon_0}$

点电荷：$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \vec{e_r}$

$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2} \vec{e_r}$

$dq: \begin{cases} \lambda dl \\ \sigma ds \\ \rho dv \end{cases}$

无限长均匀带电直线：

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 a}}$$

带电圆环：

$$\boxed{E = \frac{qx}{4\pi\varepsilon_0 (x^2+R^2)^{\frac{3}{2}}}} \xRightarrow{x \gg R} E = \frac{q}{4\pi\varepsilon_0 x^2}$$

带电圆盘：

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right) \xRightarrow{x \gg R} E = \frac{q}{4\pi\varepsilon_0 x^2}$$

$\Downarrow$ $R \gg x$，平面

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

方法：微元分割
代入模型
积分求解

静电场的高斯定理

高斯面对称 $\left\{\begin{array}{l}\text{球}\\\text{轴}\\\text{面}\end{array}\right.$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\cdot\vec{e}_r$$

$$\Psi_E = \oiint_E \vec{E}\cdot d\vec{S} = \oiint_E \frac{q}{4\pi\varepsilon_0 r^2}\cdot dS = \frac{q}{\varepsilon_0}$$

高斯定理：

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\sum_i q_i$$

E通量与孤立源的关系

对于连续带电体：

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\int dq$$

应用：

无限长带电直线：λ

$$\frac{\lambda\cdot h}{\varepsilon_0} = 2\pi r h\cdot E$$

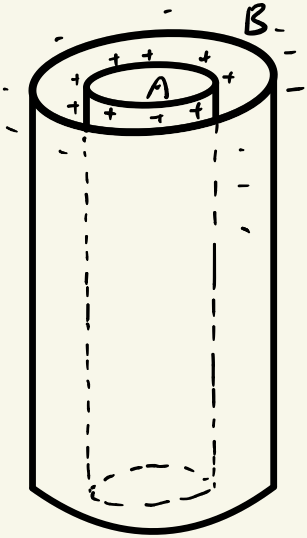
$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

平面：σ

$$\frac{\pi R^2\sigma}{\varepsilon_0} = 2\pi R^2\cdot E$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

例题：



单位高 A → $+\lambda$

B → $-\lambda$

E_{AB} ?

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ → B 静电屏蔽

静电场力的功 电势

$$A_{ab} = \int_a^b \vec{F}\cdot d\vec{l} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$ → 保守力做功

静电场的环路定理:

环流

$$A = q_0\oint_L \vec{E}\cdot d\vec{l} = 0$$ → 有源保守力场，无旋场

故 $$\oint_L \vec{E}\cdot d\vec{l} = 0$$ → 本质是电势

电势:

$$V_a = \frac{W_a}{q_0} = \int_a^{\infty}\vec{E}\cdot d\vec{l}$$

$$A_{ab} = q_0(V_a - V_b)$$

电势计算:

点电荷:

$$V_p = \frac{q}{4\pi\varepsilon_0 r}$$

连续分布:

$$V_p = \int \frac{dq}{4\pi\varepsilon_0 r}$$

带电圆环:

$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,dl}{2\pi R r}$$

$$\therefore \begin{cases} V = \int dV = \int_0^{2\pi R} \frac{q\,dl}{8\pi^2\varepsilon_0 R r} = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+x^2}} \\ V = \int_x^{\infty} \vec{E}\cdot d\vec{l} = \int_x^{\infty} \frac{qx}{4\pi\varepsilon_0 (x^2+R^2)^{\frac{3}{2}}}\,dx = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+x^2}} \end{cases}$$

$$V_p = \frac{q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}$$

$$dV = \frac{dq}{4\pi\varepsilon_0 r} \qquad U = \int_0^q \frac{dq}{4\pi\varepsilon_0 r}$$

带电圆盘:

$$dq = 2\pi r\sigma\,dr$$

$$\therefore dV = \frac{dq}{4\pi\varepsilon_0 r}$$

$$V_p = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$$

用 dU 算! 变 $dq = \begin{cases} 2\pi r\sigma\,dr \\ 4\pi r^2\rho\,dr \end{cases}$

△ 对于有限电荷才可选无穷远处电势为 0,即 $U_\infty = 0$!

静电场中的导体

静电平衡：受外电场激发，导体内部产生感应电荷

必要条件：导体内任一点的电场强度都等于零

☆ 当带电导体处于静电平衡状态时，导体内部处处没有净电荷存在，电荷只能分布于导体的外表面.

由高斯定理：

$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{S} = E\Delta S = \frac{\sigma\Delta S}{\varepsilon_0}$$

即

$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{e}_n}$$

电容器中的电容

$$C = \frac{Q}{U} \longrightarrow \text{广义电容定义式}$$

$$\varepsilon_r = \frac{C}{C_0} \longrightarrow \text{相对介电常量}$$

电容决定式：

(1) 平行板电容器

$$C = \frac{q}{\Delta V} = \frac{\sigma S}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 S}{d} \quad \text{即} \quad \boxed{C = \frac{\varepsilon_0 S}{d}}$$

(2) 圆柱形电容器.

$$\vec{E} = \frac{\lambda}{2\pi \varepsilon_0 r} \vec{e}_r$$

$$\Delta V = \int_{R_A}^{R_B} \vec{E} \cdot d\vec{r} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R_B}{R_A}$$

$$C = \frac{q}{\Delta V} = \frac{\lambda l}{\frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R_B}{R_A}} = \frac{2\pi \varepsilon_0 l}{\ln \frac{R_B}{R_A}}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln \frac{R_B}{R_A}}}$$

$$\boxed{C_l = \frac{2\pi\varepsilon_0}{\ln \frac{R_B}{R_A}}} \quad \text{(单位长度)}$$

(3) 球形电容器

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \vec{e}_r$$

$$\Delta V = \int_{R_A}^{R_B} \vec{E} \cdot d\vec{r} = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{R_A} - \frac{1}{R_B} \right)$$

$$C = \frac{q}{\Delta V} = 4\pi\varepsilon_0 \frac{R_A R_B}{R_B - R_A}$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{R_A R_B}{R_B - R_A}}$$

$$\boxed{C = 4\pi\varepsilon_0 R} \quad \text{(孤立导体球)}$$

$R_B = +\infty$

电容器串并联：

串联： $\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i}$

并联： $C = \sum_{i=1}^{n} C_i$

☆ 求电容：

1. 极板电量 Q

2. $E = ?$

3. $\Delta U = \int \vec{E} \cdot d\vec{l}$

4. $C = \frac{Q}{\Delta U}$

静电场中的电介质

电极化强度：$$P = \frac{\sum \vec{p}_e}{\Delta V} = \frac{\sigma' \Delta S l}{\Delta S l} = \sigma'$$

电极化强度（P矢量）

$$\sigma' = \vec{P} \cdot \vec{e}_n = P_n$$

介质极化所产生的极化电荷面密度等于电极化强度沿介质表面外法线的分量.

$$\vec{E} = \vec{E}_0 + \vec{E}'$$

极化电荷产生的场强

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

对于平行板电容器：

$$E = E_0 - E' = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0} = E_0 - \chi_e E$$

$$\therefore E = \frac{E_0}{1+\chi_e} \qquad U = Ed = \frac{\sigma_0 d}{\varepsilon_0 (1+\chi_e)}$$

$$C = \frac{q}{U} = (1+\chi_e) C_0$$ 即 $\boxed{\varepsilon_r = 1+\chi_e}$

电容器充满电介质后电容增大.

有电介质情况　电位移

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}(\sigma_0 S_1 - \sigma' S_2) = \frac{1}{\varepsilon_0}\sigma_0 S_1 - \frac{1}{\varepsilon_0}\oiint_S \vec{P}\cdot d\vec{S}$$

即

$$\boxed{\oiint_S (\varepsilon_0 \vec{E} + \vec{P})\cdot d\vec{S} = q_0}$$

定义**电位移**：

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$ → 类似于"光程"

$$\boxed{\oiint_S \vec{D}\cdot d\vec{S} = q_0}$$ → 有电介质时的高斯定理

电通量：

垂直于电位移线的单位面积上通过的电位移线数目.

静电场的能量

带电电容器的静电能:

$$W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q(V_1 - V_2)$$

对于平行板电容器: $W_e = \frac{1}{2}\varepsilon E^2 V$

电场中每单位体积的能量，即电场能量密度

$$\boxed{w_e = \frac{1}{2}\vec{D}\cdot\vec{E}}$$

$\longrightarrow$ 在各向同性线性介质中，$\vec{D} = \varepsilon\vec{E}$

$$w_e = \frac{1}{2}\varepsilon E^2.$$

$$W_e = \int_V w_e \, dV = \int_V \frac{1}{2}\vec{D}\cdot\vec{E} \, dV$$

大物期中复习(概念+推导)

$\boxed{\text{运动学}}$

$$\vec{r}=\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}+z(t)\vec{k}$$

$$\Delta\vec{r}=\Delta x\vec{i}+\Delta y\vec{j}+\Delta z\vec{k}\Rightarrow|\Delta\vec{r}|=\sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z^2)}$$

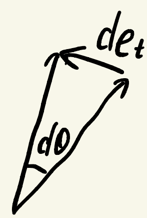
速度：$\vec{v}=\dfrac{d\vec{r}}{dt}=\dfrac{dx}{dt}\vec{i}+\dfrac{dy}{dt}\vec{j}+\dfrac{dz}{dt}\vec{k}$

速率：$v=|\vec{v}|=\sqrt{v_x^2+v_y^2+v_z^2}=\sqrt{\left(\dfrac{dx}{dt}\right)^2+\left(\dfrac{dy}{dt}\right)^2+\left(\dfrac{dz}{dt}\right)^2}$.

加速度：$\vec{a}=\dfrac{d\vec{v}}{dt}=\dfrac{d^2\vec{r}}{dt^2}=\dfrac{d^2x}{dt^2}\vec{i}+\dfrac{d^2y}{dt^2}\vec{j}+\dfrac{d^2z}{dt^2}\vec{k}$

自然坐标系下：

$$\vec{v}=v\cdot\vec{e}_t \qquad \vec{a}=\frac{d}{dt}(v\vec{e}_t)=\frac{dv}{dt}\vec{e}_t+v\frac{d\vec{e}_t}{dt}$$

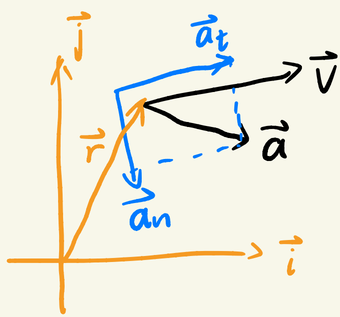


$$\frac{d\vec{e}_t}{dt}=\frac{d\theta}{dt}\vec{e}_n=\frac{d(R\theta)}{Rdt}\vec{e}_n=\frac{ds}{Rdt}\vec{e}_n=\frac{v}{R}\vec{e}_n$$

$$\therefore\ \vec{a}=\underbrace{\frac{dv}{dt}\vec{e}_t}_{\vec{a}_t}+\underbrace{\frac{v^2}{R}\vec{e}_n}_{\vec{a}_n} \qquad |\vec{a}|=\sqrt{a_t^2+a_n^2}=\sqrt{\left(\frac{dv}{dt}\right)^2+\left(\frac{v^2}{R}\right)^2}$$

转换：

$$\vec{a}_t=\frac{d|\vec{v}|}{dt}\vec{e}_t=\frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$$ ☆



圆周运动：

$\theta = \theta(t)$ 右手螺旋.

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}, \quad \vec{\beta} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$$

$$\vec{v} = \vec{\omega} \times \vec{r} \qquad v = |\vec{v}| = \frac{ds}{dt} = r\omega$$

$$\vec{a}\begin{cases} \vec{a}_t = \vec{\beta} \times \vec{r} \\ \vec{a}_n = \vec{\omega} \times \vec{v} \end{cases} \qquad \begin{cases} a_t = |\vec{a}_t| = \dfrac{dv}{dt} = r\beta \\ a_n = |\vec{a}_n| = \dfrac{v^2}{r} = r\omega^2 \end{cases} \qquad \vec{a} = \vec{\beta} \times \vec{r} + \vec{\omega} \times \vec{v}.$$

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\theta = \omega t$	$v = \frac{ds}{dt}$ $a = \frac{dv}{dt}$ $s = vt$
$\omega = \omega_0 + \beta t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\beta t^2$ $\omega^2 - \omega_0^2 = 2\beta(\theta - \theta_0)$	$v = v_0 + at$ $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 - v_0^2 = 2a(x - x_0)$

动力学

质心：$\vec{r}_c = \dfrac{\int \vec{r}\,dm}{m}$

动量：$\vec{F} = m\dfrac{d\vec{v}}{dt} = \dfrac{d\vec{p}}{dt}$ $\quad \therefore \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v_1} - m\vec{v_2}$

动量守恒定理：$\sum F_i = 0$ 时，$\sum m_i \vec{v_i}$ = 常矢量.（同一参考系下）

角动量：$\vec{L} = \vec{r} \times \vec{p}$

角动量定理：$\dfrac{d\vec{L}}{dt} = \vec{r} \times \dfrac{d\vec{p}}{dt}$ $\quad \therefore \vec{M} = \vec{r} \times \vec{F} = \dfrac{d\vec{L}}{dt}$

角动量守恒定理：$\vec{L}$ = 常量（$\vec{M} = \vec{0}$）

功：$A = \int_a^b \vec{F} \cdot d\vec{r}$

动能：$E_K = \frac{1}{2}mv^2$ $\quad$ 动能定理：$A = E_{K_1} - E_{K_2}$

保守力：$\oint_L \vec{F} \cdot d\vec{r} = 0$

刚体定轴转动

力矩：$\vec{M}=\vec{r}\times\vec{F}$

角速度矢量：$\vec{v}=\vec{\omega}\times\vec{r}$

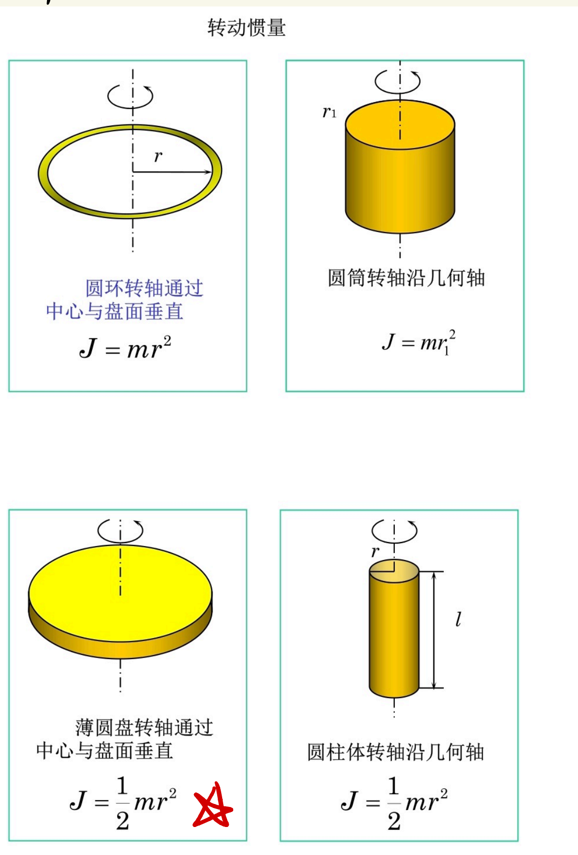
定轴转动定律：$\underline{M_z=J\alpha=J\frac{d\omega}{dt}}$

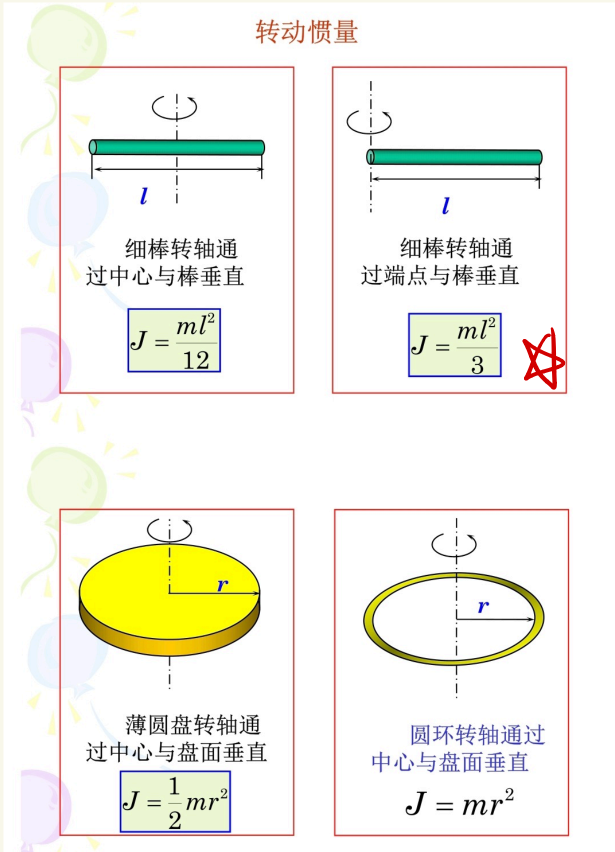
转动惯量：$\underline{J=\int r^2dm}$

平行轴定理：$\underline{J=J_c+Md^2}$

垂直轴定理：$J_z=J_x+J_y$

常见转动惯量：





球壳：$J=\frac{2}{3}mr^2$　　厚壁圆筒：$J=\frac{1}{2}m(R_1^2+R_2^2)$

球体：$J=\frac{2}{5}mr^2$　　圆柱体：$J=\frac{1}{4}mr^2+\frac{1}{12}ml^2$

刚体定轴转动定律： $M = J\beta$

力矩的功： $A = \int_{\theta_0}^{\theta} M d\theta$.　　力矩的功率： $P = M\omega$.

刚体转动动能： $E_k = \frac{1}{2} J\omega^2$　　动能定理： $A = \int_{\theta_0}^{\theta} M d\theta = \frac{1}{2} J\omega^2 - \frac{1}{2} J\omega_0^2$.

与角动量关系： $E_k = \frac{L^2}{2J}$.

刚体的角动量： $L_z = J\omega$　　定轴转动定律： $M_z = \frac{dL_z}{dt}$

角动量定理： $\int_0^t M_z dt = J\omega - J_0\omega_0$

体系对比

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $\theta = \omega t$	$v = \frac{ds}{dt}$ $a = \frac{dv}{dt}$ $s = vt$
$\omega = \omega_0 + \beta t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\beta t^2$ $\omega^2 - \omega_0^2 = 2\beta(\theta - \theta_0)$	$v = v_0 + at$ $x = x_0 + v_0 t + \frac{1}{2} a t^2$ $v^2 - v_0^2 = 2a(x - x_0)$
$M = J\alpha$ $Mt = J\omega - J_0\omega_0$ $\sum J\omega = C$	$F = ma$ $Ft = mv - mv_0$ $\sum mv = C$
$E_k = \frac{1}{2} J\omega^2$ $A = M\theta$ $M\theta = \frac{1}{2} J\omega^2 - \frac{1}{2} J\omega_0^2$	$E_k = \frac{1}{2} m v^2$ $A = Fs$ $Fs = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$

静电场

稳恒磁场.

电流:

定义: $I = \dfrac{dq}{dt} = neSv_d$ → 决定式

电流密度: $j = \dfrac{dI}{dS_\perp} = nev_d$

是空间位置的矢量函数,精确描述导体中电流分布情况.

$\vec{j}$ 矢量,方向与该点正电荷运动的方向一致.

$$\boxed{I = \int_S \vec{j} \cdot d\vec{S}}$$

$\vec{j} = \gamma \vec{E}$,其中 $\gamma = \dfrac{1}{\rho} = \dfrac{l}{RS}$.

△电流的连续性方程:

$$\boxed{\oint_S \vec{j} \cdot d\vec{S} = -\frac{dq}{dt}} \Rightarrow \oint_S \vec{j}\, d\vec{S} = 0$$ (稳恒电流)

描述散度

电动势:

非静电力:能不断分离正负电荷使正电荷逆静电场力运动

△定义:单位正电荷绕闭合回路运动一周,非静电力所做的功.

$$W = \oint_l q\vec{E}_k \cdot d\vec{l}$$

$$\boxed{\varepsilon = \frac{dA}{dq} = \oint_L \vec{E}_k \cdot d\vec{l}}.$$

"非静电性场的场强",沿整个闭合电路的环流不等于零.

磁感应强度

定义式：$B = \frac{F_m}{qv}$.

磁感应线和磁通量

△在任何磁场中，每一条磁感应线都是闭合的，且与电流方向成右旋关系.

磁通量：$d\Phi = \vec{B} \cdot d\vec{S}$ 单位为 Wb（韦伯）

$$\Phi = \int_S \vec{B} \cdot d\vec{S}.$$

毕奥-萨伐尔定律：

电流元：$I d\vec{l}$.

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \vec{e}_r}{r^2}}$$

$$\vec{B} = \int_L d\vec{B} = \frac{\mu_0}{4\pi} \int_L \frac{I d\vec{l} \times \vec{e}_r}{r^2}.$$

应用：

△载流长直导线

$$B = \frac{\mu_0 I}{4\pi a}(\sin\beta_1 - \sin\beta_2)$$

☆无限长

$$\boxed{B = \frac{\mu_0 I}{2\pi a}}$$

△载流圆线圈（轴线）

$$B = \frac{\mu_0 I}{2\pi} \cdot \frac{S}{(R^2+x^2)^{\frac{3}{2}}}$$

推导：

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{r^2} dl.$$

$$r = \sqrt{R^2+x^2}.$$

$$\therefore B = 2\pi R \cdot dB \sin\theta$$

☆ $x=0$ 处（正中心）

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

☆ $x \gg R$ 处（无穷远）

$$\boxed{B = \frac{\mu_0 IS}{2\pi x^3}}$$

△载流直螺线管

☆ $\boxed{B = \mu_0 n I}$ $(l \gg d)$

△运动电荷的磁场

$$I = qnvS$$

$$\vec{B}_q = \frac{dB}{dn} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \vec{e}_r}{r^2}$$

恒定磁场的高斯定理

通过任一闭合曲面的总磁通量总等于零.

$$\boxed{\oiint_S \vec{B} \cdot d\vec{S} = 0}$$

→ 无源场

安培环路定理

$$\boxed{\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum I}$$

→ 有旋场

洛伦兹力

$$\vec{F} = q\vec{v} \times \vec{B}.$$

Δ 均匀磁场运动

$$R = \frac{mv}{qB} \qquad T = \frac{2\pi m}{Bq}$$

霍尔效应

$$U = R_H \frac{IB}{d}$$

$$R_H = \frac{1}{nq}$$

磁场对载流导线的作用

△ 安培定律：

$$\vec{F} = \int_L d\vec{F} = \int_L I d\vec{l} \times \vec{B}.$$

化曲为直

(类似保守力作功的感觉？)

△ 对载流线圈：

$$F = NBIl\sin(\omega t+\varphi).$$

$$M = NBIS\sin(\omega t+\varphi)$$

磁场力的功

$$dA = Id\Phi$$

$$\boxed{A = \int I d\Phi = I\Delta\Phi}$$

有磁介质时，引入磁场强度 $\vec{H}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

故有 $\oint \vec{H}\cdot d\vec{l} = \sum I$

△ 对于真空：

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0}}$$

Summary

核心：$B = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot d\vec{l} \times \vec{e}_r}{r^2}$.

$\oiint_S \vec{B} \cdot d\vec{S} = 0$ （散度）

$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \sum I$. （旋度）

电磁感应

现象

当穿过一个闭合导体回路所包围的面积内的磁通量发生变化时，在导体回路就会产生感应电流.

楞次定律 → "惯性".

法拉第电磁感应定律:

$$\boxed{\varepsilon_i = -\frac{d\Phi}{dt}}$$

→ 对于N匝线圈 $\varepsilon_i = -N\frac{d\Phi}{dt} = -\frac{dN\Phi}{dt}$.

△ 若闭合回路电阻为R，感应电流

$$I_i = -\frac{1}{R}\frac{d\Phi}{dt}$$

$$q = \left|\int_{t_1}^{t_2} I\,dt\right| = \frac{1}{R}\left|\Phi_2 - \Phi_1\right|.$$

☆ 方向问题:

① 任意规定回路的绕行正方向，电动势同向

② 根据 ε_i 的正负判断 ε_i 的方向.

积分形式:

$$\boxed{\varepsilon_i = \oint_L \vec{E}_K \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}}.$$

动生电动势

△ 磁场中运动的导线：

$$\boxed{\varepsilon_i = -\frac{d\Phi}{dt} = -Blv}$$ → 切割磁感应线产生

对于微观电子理论：

$Fe = Fm$

$$d\varepsilon_i = \vec{E}_K \cdot d\vec{l} = \vec{v} \times \vec{B} \cdot d\vec{l}$$

$-e \cdot \vec{E}_K = -e\vec{v} \times \vec{B}$

$$\boxed{\varepsilon_i = \int_L \vec{v} \times \vec{B} \cdot d\vec{l}}$$

△ 转动线圈：

$$\Phi = BS\cos\theta$$

$$\varepsilon_i = -N\frac{d\Phi}{dt} = NBS\sin\theta\frac{d\theta}{dt} \xrightarrow{\theta = \omega t} \text{即 } \varepsilon_i = NBS\omega\sin\omega t$$

$$\boxed{\varepsilon_i = \varepsilon_0 \sin\omega t}$$ (交变电动势)

感生电动势

变化的磁场在其周围激发了一种电场. → 感生电场 → 非保守力场

$$\oint_L \vec{E}_i \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} .$$

△注意: $\vec{E}$ 与 $\frac{\partial \vec{B}}{\partial t}$ 成左旋关系

△螺线管内外:

$$E_i = \begin{cases} -\dfrac{r}{2}\dfrac{dB}{dt}, & r<R \\ -\dfrac{R^2}{2r}\dfrac{dB}{dt}, & r>R \end{cases}$$

本质: $\varepsilon_i = -\dfrac{d\Phi}{dt}$

$$\oint_L \vec{E}_i \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{S}$$

△螺线管内金属棒:

$$\varepsilon_i = -\frac{d\Phi}{dt} = S_{\Delta Ol} \cdot \frac{dB}{dt}$$

等势线可视为导线, 构成闭合回路.

自感应与互感应

自感电动势:

$$\boxed{\varepsilon_L = -L\frac{dI}{dt}}$$

$$\left(L = \frac{\Phi_N}{I} \right)$$

单位: H(亨利)

△同轴电缆:

$$\boxed{L = \frac{\mu_0}{2\pi}\ln\frac{R_2}{R_1}}$$

计算步骤:

① 设电流 I, 求激发的 B

② 求磁通链 Φ_N.

③ 由定义求出自感系数 L

互感电动势：

$$\mathcal{E}_{21} = -M\frac{dI_1}{dt}, \quad \mathcal{E}_{12} = -M\frac{dI_2}{dt} \quad \left(M = \frac{\Phi_{21}}{I_1} = \frac{\Phi_{12}}{I_2} \right)$$

（2在1影响下的ε）（1在2影响下的ε）.

耦合：

$$M = k\sqrt{L_1 L_2}$$

计算步骤：

① 假设1中电流I_1，求激发磁场B_1.

② 求2中磁通链Φ_{21}，

③ 由定义求M.

磁场的能量

$$\boxed{W_m = \frac{1}{2} L I_0^2}$$

磁场能量密度：

$$\boxed{w_m = \frac{1}{2} \vec{B} \cdot \vec{H}}$$

$$w_m = \frac{1}{2} \frac{B^2}{\mu_0}$$

故有 $dW_m = w_m dV = \frac{1}{2} \vec{B} \cdot \vec{H} dV$.

$$\boxed{W_m = \frac{1}{2} \int_V \vec{B} \cdot \vec{H} dV.}$$

狭义相对论

S：静止

S'：运动

$\boxed{\text{洛伦兹变换}}$

$S \longrightarrow S'$ 正

$$\boxed{\begin{cases} x' = \dfrac{x-ut}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \\ y' = y \\ z' = z \\ t' = \dfrac{t-\frac{ux}{c^2}}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \end{cases}}$$

$S' \longrightarrow S$ 逆

$$\boxed{\begin{cases} x = \dfrac{x'+ut'}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \\ y = y' \\ z = z' \\ t = \dfrac{t'+\frac{ux'}{c^2}}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \end{cases}}$$

相对论因子：$\gamma = \dfrac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} > 1$

△ 同时，同地 $\Rightarrow$ 其他惯性系中才同时，

长度收缩：

$$l' = l_0\sqrt{1-\left(\frac{u}{c}\right)^2}. \Rightarrow \Delta x = \frac{\Delta(x'+ut')}{\sqrt{1-\beta^2}} \quad \boxed{l_0 = \gamma l'}$$

时间延缓：

$$t = \frac{t_0}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \Rightarrow \Delta t' = \frac{\Delta\left(t-\frac{ux}{c^2}\right)}{\sqrt{1-\beta^2}} \quad \boxed{\Delta t' = \gamma \Delta t}$$